Students’ Overuse of Proportionality on Missing-Value Problems: How Numbers May Change Solutions
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[The research for the article was based on work with 508 students from Gr. 4-6, but reference is made to students in middle and secondary schools. The problems were also used with pre-service elementary teachers.]

Summary:
Students have a tendency to over-proportionalize whether a missing-value problem is a proportion problem or not, i.e. they tend to use multiplying or dividing to solve all missing value problems. Part of the difficulty is that without a solid number sense, students tend to take their cues from the numbers in the problem. For example, they will base their decision about what operation to use based on the type of numbers in the problem, and/or the number of numbers in the problem. The research used several categories of problems: directly proportional, partially proportional, non-proportional – additive, non-proportional – constant, non-proportional – other.

Some implications for teaching:
- use numbers that are “not nice” as well as numbers that are “nice”
- include more information than is actually needed to solve the problem
- make explicit the difference between directly proportional, partially proportional, and non-proportion problems, i.e, additive vs. multiplicative vs. constant relationships; number sense – meaning of mult. and div. with integers and non-integers
- analyze correct and incorrect solution methods
- use more authentic problems

Examples of Problems Students Solve Incorrectly:
1. John’s best time to run 100m is 17 sec. How long will it take him to run 1 km?
   Student answer: The real-world situation evoked by this problem allows no single, precise answer, but most students answered 170 sec. without thinking about the context.

2. Sue and Julie are running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many had Sue run?
   Student answer: The tendency is to see all problems like this as multiplicative relationships and answer either 45 or 5. The correct answer is 21.

3. Farmer Gus needs 8 hours to fertilize a square pasture with sides of 200m. How much time will he approximately need to fertilize a square pasture with sides of 600m?
   Student answer: Again, students tend to see this as a multiplicative relationship and answer 24 hours, instead of seeing that area needs to be considered. The correct answer is 72 hours.

4. Calculate the cost of 0.820 kg of cheese at $12/kg. Calculate the cost of 1.240 kg of cheese at $12/kg.
   Student answer: Tendency to use division to find the first answer and multiplication to find the second. This seems to be related to an incorrect sense that multiplication makes bigger, division makes smaller. The correct answers are $9.84 and $14.88.

5. If 25 musicians need 50 minutes to play a piece of music, how much time will 75 musicians need to play the same piece of music?
   Student answer: Tendency to see a multiplicative relationship here and answer 150 min., not seeing that the relationship is constant. The correct answer is 50 min.

6. The locomotive of a train is 12m long. If there are 4 carriages connected to the locomotive, the train is 44m long in total. If there were 8 carriages connected to the locomotive, how long would the train be?
   Student answer: There is a proportional relation between two variables, but a constant element needs to be taken into account as well. Students tend to answer 88m instead of 76m.